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## Coherency Strain-Assisted Equilibrium Segregation at Heterophase Interfaces

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**Abstract.** Theoretical work in the field of interfacial segregation is focused mainly on the sharp interface approach beginning with the Gibbs' introduction of the dividing surface. In this paper a continuum approach to the problem of interfacial segregation, which deals with continuous distributions of quantities at interfaces, is developed. The current study concentrates on heterophase interfaces in alloys as opposed to grain boundaries or antiphase domain boundaries. The important effect of coherency strains is taken into account. The Gibbs adsorption theorem is discussed in the framework of a continuum approach. A comparison is made between the sharp (Gibbsian) and diffuse interface approaches.

**Keywords:** interfacial segregation, theory, coherency strain

### 1. Introduction

The theoretical understanding of microstructural evolution plays an important role in the control of the physical and mechanical properties of technologically significant new complex materials. Heterophase metal alloys, which structurally consist of two or more phases of the same material, show promising results for aerospace applications (Al-Sc-Mg) and the electronics industry (Cu-Sn-Bi). A crucial aspect of such materials is the interface between the matrix and a precipitate of another phase, a high number density of which (up to  $10^{18} \text{ cm}^{-3}$ ) may develop during materials processing. Interfaces may appear as a result of different transformations in materials: (i) grain boundaries—misorientation of growing crystals; (ii) isomorphous transformations that occur by the mechanism of spinodal decomposition; or (iii) polymorphic transformations that produce interfaces separating phases of different crystalline symmetry—heterophase interfaces. There is a substantial body of theoretical, simulation and experimental work devoted to segregation at a free surface and grain boundary. Our research, however, deals with segregation of a minority component at well-defined *heterophase* interfaces in multicom-

ponent alloys. Another reason for studying this subject is the emergence of a great number of simulation results where interfacial segregation is observed, but the driving force for segregation is difficult to identify.

A change of crystalline symmetry is usually accompanied by the development of misfit strain in materials as a result of the different lattice spacings in different phases. Usually small precipitates grow from the bulk without loss of coherency on their boundaries. Thus it is necessary to account for the effect of coherency strain on the heterophase segregation.

Interfacial segregation was first discussed by Gibbs [1], who viewed an interface as a transition layer and introduced the concept of a dividing surface to describe the adsorption on this layer. According to Gibbs, the surface energy is the excess of the appropriate thermodynamic potential in the material due to the presence of an interface between two contiguous phases. To find the relationship between the surface energy and mass quantities associated with an interface, Gibbs introduced a dividing surface as a concentrator of all heterogeneities in the system. The concept of the dividing surface allowed Gibbs to establish relationships where the mass quantities depend on the position of

that surface, because equilibrium densities of species are not equal in adjacent phases. Notice, however, that the definition of the surface energy is independent of the position of the dividing surface because the thermodynamic potential must be equal in both bulk phases at equilibrium. Gibbs identified the relationship between surface thermodynamic quantities, like pressure, chemical potential, surface entropy, as the cause of this conundrum and suggested placing the dividing surface such that the mass quantity of one species vanishes. The latter eliminates a superficial degree of freedom of an interface at the expense of introducing artificial quantities that are difficult to measure. The ambiguity of the Gibbsian excess was thoroughly studied and discussed by Cahn [2] and Balluffi and Sutton [3]. Voorhees and Johnson included the coherency strain effect into the Gibbsian formulation [4].

Recently the author has developed a thermodynamically consistent continuum approach to the problem of interfacial segregation [5]. An advantage of the continuum method is the possibility to deal with the measurable quantities, while avoiding the cumbersome dividing surface construct. The Gibbs adsorption theorem was discussed in the framework of a continuum approach. A comparison was made between the sharp (Gibbsian) and diffuse interface approaches. The expressions for the adsorption quantities were obtained and segregation at homophase and heterophase interfaces in thermodynamically linear systems were calculated. The principal thrust of the current project is to develop a consistent continuum approach to the problem of segregation and incorporate the coherency strain into the continuum model of microstructural evolution.

## 2. Thermodynamics

### 2.1. *The Free Energy of the System*

The first step in the theoretical description is the selection of a set of independent variables that specify the system. A phase transformation can occur as a result of a change of the thermodynamic variables of a system, among which temperature,  $T$ , is the most common cause. Another set of thermodynamic variables that affect a transformation are concentrations of different components,  $c_1, c_2, c_3, \dots$ , that specify the overall composition of an alloy.

The conditions of thermodynamic equilibrium in a two-phase solid also depend on nonhydrostatic stresses that develop in the material as a result of external load-

ing or, more importantly for the present treatment, the condition of coherency of phases at an interface. The latter implies that the lattice planes are continuous across the interface, which causes coherency strains in the crystal if the lattice spacings in the two phases are not identical. Thus the strain tensor,  $\varepsilon_{ij}$ , must be an independent thermodynamic degree of freedom.

Fundamental differences between the initial ( $\alpha$ ) and final ( $\beta$ ) phases of a polymorphic transformation are their symmetries: (i) solid-liquid; (ii) fcc-bcc; (iii) austenite-martensite; or (iv) order-disorder transition of the first kind. According to Landau theory [6], a phase transition may be characterized by one or more continuous variables  $\eta_l$ , usually called order parameters, which take on specified values in the  $\alpha$  and  $\beta$  phases.

It is not of a stretch to say that all thermodynamic properties of a material are determined by its free energy. In the framework of the continuum approach the free energy of the material becomes a continuous function of the variables:  $F = F(T, c_k, \varepsilon_{ij}, \eta_l)$ . The presence of precipitates of a second phase and the appearance of heterophase interfaces make our system essentially inhomogeneous; that is, there appear gradients of some of the independent variables discussed above, e.g., composition, strain, and the order parameter. This affects the total free energy in two ways. Firstly, the total free energy becomes a functional over the entire system:

$$F = \int \hat{f} d^3x. \quad (1)$$

where the integrand  $\hat{f}$  is the free energy density. Secondly, the free energy density becomes a function of the gradients of the thermodynamic variables as well as the variables themselves. Landau considered small heterogeneities in a system and introduced the square-gradient approximation for the order parameter variation [7]. Cahn and Hilliard considered an expansion of the free energy with respect to the concentration gradients and derived a square-gradient approximation for the concentration portion of the free energy [8]. The problem of the strain gradients was analyzed in Refs. [9, 10].

### 2.2. *Equations of Equilibrium*

Conditions of equilibrium of such a system require minimization of the free energy, Eq. (1), with respect

to distributions of the strain, concentration and order parameter, provided the overall concentration of the alloy,  $\{C\}$ , remains constant and the stress on the surface of a sample is  $\Sigma_{ik}^0$ . The equation of equilibrium of a deformed body may be represented as follows [11]:

$$\partial_{x_k} \Sigma_{ik} = 0; \quad (2a)$$

where the stress tensor is:

$$\Sigma_{ik} \equiv \left( \frac{\delta F}{\delta \varepsilon_{ik}} \right)_{T, c_k, \eta_l}. \quad (2b)$$

Minimization of the free energy, Eq. (1), with respect to the concentration distribution, constrained by the condition of constant overall composition  $\{C\}$ , usually is accomplished with the aid of Lagrange multipliers,  $\lambda_k$ :

$$\left( \frac{\delta F}{\delta c_k} \right)_{T, \varepsilon_{ij}, \eta_l} = \lambda_k. \quad (3)$$

Here the Lagrange multipliers,  $\lambda_k$ , are the chemical potential differences of the species and are called diffusion potentials [10].

Order parameters are free of the constraints associated with other thermodynamic variables, for instance, concentration or strain. That is why the free energy minimum with respect to the order parameter distributions are achieved when:

$$\left( \frac{\delta F}{\delta \eta_l} \right)_{T, \varepsilon_{ij}, c_k} = 0. \quad (4)$$

A flat interface corresponds to a one-dimensional (1D) distribution of variables. For simplicity we consider a unidirectional state of stress-strain in a binary alloy with the one-component order parameter at constant temperature. Thus the free energy of a heterogeneous system with an interface may be represented by:

$$F = A \int_{\alpha}^{\beta} \left[ f(T, c, \varepsilon, \eta) + \frac{1}{2} \kappa_{\eta} (\nabla \eta)^2 + \frac{1}{2} \kappa_{\varepsilon} (\nabla \varepsilon)^2 + \frac{1}{2} \kappa_c (\nabla c)^2 \right] dx; \quad (5)$$

where  $A$  is the surface area of the interface and integration spreads on the region of heterogeneity in the material, that is the interface, and on the contiguous bulk phases. Then the conditions of mechanical (2),

chemical (3), and phase (4) equilibrium may be rewritten as follows:

$$\kappa_{\eta} \frac{d^2 \eta}{dx^2} = \partial_{\eta} f; \quad (6a)$$

$$\kappa_c \frac{d^2 c}{dx^2} = \partial_c f - \lambda; \quad (6b)$$

$$\kappa_{\varepsilon} \frac{d^2 \varepsilon}{dx^2} = \partial_{\varepsilon} f - \Sigma_0. \quad (6c)$$

where  $\Sigma_0 = \Sigma_{xx}^0$ . The 1D nature of an interface allows us to integrate the equilibrium system, Eq. (6), once. The first integral is:

$$f(T, c, \varepsilon, \eta) - \lambda c - \Sigma_0 \varepsilon - \frac{1}{2} \kappa_c \left( \frac{dc}{dx} \right)^2 - \frac{1}{2} \kappa_{\varepsilon} \left( \frac{d\varepsilon}{dx} \right)^2 - \frac{1}{2} \kappa_{\eta} \left( \frac{d\eta}{dx} \right)^2 = \mu_1; \quad (6d)$$

where the constant  $\mu_1$  is the chemical potential of species 1. Existence of the first integral, Eq. (6d), means that the chemical potentials of both species are constant throughout the system at equilibrium, which corresponds to the equilibrium conditions of a two-phase system with a sharp interface.

Setting all gradients equal to zero, the equilibrium equations, Eqs. (6a)–(6c), and the first integral, Eq. (6d), allow us to establish the tie-line relation between the bulk phases far from the interface, which are exactly the same as those for the sharp interface:

$$\partial_{\eta} f(T, c_{\alpha}, \varepsilon_{\alpha}, \eta_{\alpha}) = \partial_{\eta} f(T, c_{\beta}, \varepsilon_{\beta}, \eta_{\beta}) = 0; \quad (7a)$$

$$\partial_c f(T, c_{\alpha}, \varepsilon_{\alpha}, \eta_{\alpha}) = \partial_c f(T, c_{\beta}, \varepsilon_{\beta}, \eta_{\beta}) = \lambda; \quad (7b)$$

$$\partial_{\varepsilon} f(T, c_{\alpha}, \varepsilon_{\alpha}, \eta_{\alpha}) = \partial_{\varepsilon} f(T, c_{\beta}, \varepsilon_{\beta}, \eta_{\beta}) = \Sigma_0; \quad (7c)$$

$$f(T, c_{\alpha}, \varepsilon_{\alpha}, \eta_{\alpha}) - \lambda c_{\alpha} - \Sigma_0 \varepsilon_{\alpha} = f(T, c_{\beta}, \varepsilon_{\beta}, \eta_{\beta}) - \lambda c_{\beta} - \Sigma_0 \varepsilon_{\beta} = \mu_1. \quad (7d)$$

According to Gibbs, the surface energy is the excess of the appropriate thermodynamic potential in the material due to the presence of an interface between two contiguous phases. The appropriate thermodynamic potential is:

$$\Omega = \int \int \int \left[ f(T, c, \varepsilon, \eta) - \lambda c - \Sigma_0 \varepsilon - \mu_1 + \frac{1}{2} \kappa_c (\nabla c)^2 + \frac{1}{2} \kappa_{\varepsilon} (\nabla \varepsilon)^2 + \frac{1}{2} \kappa_{\eta} (\nabla \eta)^2 \right] d^3 x. \quad (8)$$

Then the surface energy is

$$\sigma = \int_{-\infty}^{+\infty} \left[ f(T, c, \varepsilon, \eta) - \lambda c - \Sigma_0 \varepsilon - \mu_1 + \frac{1}{2} \kappa_c \left( \frac{dc}{dx} \right)^2 + \frac{1}{2} \kappa_\varepsilon \left( \frac{d\varepsilon}{dx} \right)^2 + \frac{1}{2} \kappa_\eta \left( \frac{d\eta}{dx} \right)^2 \right] dx. \quad (9a)$$

The integration in Eq. (9a) may be extended into infinities because, according to Eq. (6d), the integrand vanishes far from the interface. Utilizing an equilibrium condition, Eq. (6d), one obtains:

$$\begin{aligned} \sigma &= 2 \int_{-\infty}^{+\infty} [f(T, c, \varepsilon, \eta) - \lambda c - \Sigma_0 \varepsilon - \mu_1] dx \\ &= \int_{-\infty}^{+\infty} \left[ \kappa_c \left( \frac{dc}{dx} \right)^2 + \kappa_\varepsilon \left( \frac{d\varepsilon}{dx} \right)^2 + \kappa_\eta \left( \frac{d\eta}{dx} \right)^2 \right] dx. \end{aligned} \quad (9b)$$

### 2.3. Gibbs Adsorption Theorem

An important interfacial characteristic is the magnitude of the relative adsorption, which measures the excess of one species at the interface over another one. In Ref. [5] the author has developed a thermodynamically consistent continuum approach to the problem of interfacial segregation and derived the Gibbs adsorption theorem in the form:

$$d\sigma = -\Gamma_S^{(1)} \cdot dT - \Gamma_l^{(1)} \cdot d\mu_l; \quad (10a)$$

$$\Gamma_S^{(1)} = \int_{\alpha}^{\beta} \left\{ \hat{s} - s_\alpha - (\rho_1 - \rho_{1\alpha}) \frac{[s]}{[\rho_1]} \right\} dx; \quad (10b)$$

$$\Gamma_l^{(1)} = \int_{\alpha}^{\beta} \left\{ \rho_l - \rho_{l\alpha} - (\rho_1 - \rho_{1\alpha}) \frac{[\rho_l]}{[\rho_1]} \right\} dx. \quad (10c)$$

where  $\Gamma_S^{(1)}$  and  $\Gamma_l^{(1)}$  are relative surface entropy and adsorption. The quantities in square brackets may be called interfacial jumps and are defined as follows:

$$[\varphi] \equiv \varphi_\beta - \varphi_\alpha. \quad (11)$$

For the free energy, Eq. (5), the theorem, Eq. (10), yields:

$$\Gamma_2^{(1)} = \int_{\alpha}^{\beta} \left\{ c - c_\alpha - (\varepsilon - \varepsilon_\alpha) \frac{[c]}{[\varepsilon]} \right\} dx. \quad (12)$$

Notice that the integrand in this expression vanishes on both ends of the interface that is in the  $\alpha$  and  $\beta$  phases. It has been shown in Ref. [5] that segregation

at a heterophase interface vanishes if the free energy depends quadratically on concentration and strain (that is, a “linear” thermodynamic system). This allowed us to identify the nonlinear interactions between the species in the system as the driving force for segregation. Consequently, in the present study we analyze the influence of the nonlinear chemical and elastic interactions on interfacial segregation at heterophase interfaces.

## 3. Segregation at a Heterophase Interface

### 3.1. Alloy Solution Thermodynamics

The free energy of an alloy, where a heterophase interface is possible, was analyzed in Ref. [5] and is represented here in the form:

$$f = f_0(T) + \Delta_\eta f + \Delta_c f + \Delta_\varepsilon f + \Delta_{\varepsilon c} f; \quad (13a)$$

$$\Delta_\eta f = a \left\{ \frac{1}{2} (1 - \tau) \eta^2 - \frac{2}{3} \eta^3 + \frac{1}{4} \eta^4 \right\};$$

$$\tau = \tau_c (c - c_*) + \tau_\varepsilon \varepsilon; \quad (13b)$$

$$\Delta_c f = \frac{1}{2} L_\alpha (c - c_*)^2; \quad \Delta_\varepsilon f = \frac{1}{2} K_\alpha \varepsilon^2 \quad (13c)$$

where  $c_*$  is a characteristic concentration that corresponds to the point of the free energy minimum in an undeformed  $\alpha$ -phase,  $L_\alpha$  is the second derivative of the free energy with respect to the concentration and  $K_\alpha$  is the elastic modulus of the  $\alpha$ -phase. The function  $\tau$  describes the coupling between the order parameter, concentration and strain.

A very important source of nonlinearities, hence interfacial segregation, is the nonlinear interaction between composition and strain, e.g. nonlinear dependence of the lattice parameter on concentration. To account for that in the present treatment, we take:

$$\Delta_{\varepsilon c} f = \zeta \{ A \varepsilon (c - c_*)^2 + B \varepsilon^2 (c - c_*) \}; \quad (13d)$$

where  $\zeta$  may be called a nonlinear Vegard’s coefficient.

The conditions of equilibrium, Eqs. (6a)–(6c), for the free energy, Eq. (13), take the form:

$$\begin{aligned} L_\alpha (c - c_*) &= \lambda + \frac{1}{2} a \tau_c \eta^2 + \kappa_c c'' \\ &\quad - \zeta \{ 2A \varepsilon (c - c_*) + B \varepsilon^2 \}; \end{aligned} \quad (14a)$$

$$\begin{aligned} K_\alpha \varepsilon &= \Sigma_0 + \frac{1}{2} a \tau_\varepsilon \eta^2 + \kappa_\varepsilon \varepsilon'' - \zeta \{ A (c - c_*)^2 \\ &\quad + 2B \varepsilon (c - c_*) \}; \end{aligned} \quad (14b)$$

$$\kappa_\eta \eta'' = a \eta (\eta_* - \eta) (\eta_\beta - \eta). \quad (14c)$$

Setting  $c'' = \varepsilon'' = \eta'' = 0$ , Eqs. (14) turn into Eqs. (7a)–(7c), which can be resolved for the equilibrium values of the concentration, strain, and order parameter. For instance, for the latter one obtains:

$$\eta_\alpha = 0; \quad \eta_* = 1 - \sqrt{\tau_*}; \quad \eta_\beta = 1 + \sqrt{\tau_\beta}. \quad (15)$$

where  $\eta_*$  is a saddle point of the free energy.

### 3.2. A “Linear” Thermodynamic System

To understand better the thermodynamics of a system described by the free energy, Eqs. (13), one may choose first to look at a “linear” system that is, with  $\zeta = 0$  in Eq. (13d). For the jumps of concentration and strain between the  $\alpha$  and  $\beta$  phases in the linear system, the equilibrium Eqs. (14) yield:

$$[c] = \frac{a\tau_c}{2L_\alpha}\eta_\beta^2; \quad (16)$$

$$[\varepsilon] = \frac{a\tau_\varepsilon}{2K_\alpha}\eta_\beta^2; \quad (17)$$

Then Eq. (15) allows one to find the first tie-line relation in the form:

$$[\tau] = m(1 + \sqrt{\tau_\beta})^2. \quad (18)$$

where  $m$  is the modulus:

$$m \equiv \frac{a}{2} \left( \frac{\tau_\varepsilon^2}{K_\alpha} + \frac{\tau_c^2}{L_\alpha} \right). \quad (19)$$

Using Eqs. (16) and (17) for Eq. (7d) we obtain the second tie-line relation:

$$\eta_\beta = \frac{4/3}{1-m}. \quad (20)$$

Solving Eqs. (15), (18), and (20) simultaneously, the tie line may be resolved as follows:

$$\tau_\alpha = \frac{1/9 - m}{1 - m}; \quad \tau_\beta = \left( \frac{1/3 + m}{1 - m} \right)^2. \quad (21)$$

Notice that, although concentration and strain in both phases depend on the diffusion potential  $\lambda$  and external stress  $\Sigma_0$ ,  $\tau_\alpha$  and  $\tau_\beta$  are independent of these. Also interesting to note that an equilibrium invariant transition, that is without change of composition or strain, is possible in the system with  $m = 0$  at  $\tau_\alpha = \tau_\beta = 1/9$ .

For the results of the present study to be applicable to real materials, one has to relate parameters of the free energy, Eq. (13), to the measurable parameters of

such material. Equations (17) and (20) yield the expression for the misfit strain, i.e. the stress free ( $\Sigma_0 = 0$ ) transformation strain in the  $\beta$ -phase:

$$[\varepsilon] = \frac{8a\tau_\varepsilon}{9K_\alpha(1-m)^2}. \quad (22)$$

For the elastic modulus and the second derivative of the free energy with respect to composition in the  $\beta$ -phase Eqs. (13) and (20) yield:

$$\left( \frac{\partial^2 f}{\partial \varepsilon^2} \right)_c (c_\beta, \varepsilon_\beta, \eta_\beta) \equiv K_\beta = K_\alpha - \frac{2a\tau_\varepsilon^2}{1+3m}; \quad (23)$$

$$\left( \frac{\partial^2 f}{\partial c^2} \right)_\varepsilon (c_\beta, \varepsilon_\beta, \eta_\beta) \equiv L_\beta = L_\alpha - \frac{2a\tau_c^2}{1+3m}; \quad (24)$$

Equations (19), (23) and (24) allow to express the parameters  $\tau_c$  and  $\tau_\varepsilon$  through measurable materials properties and yield constraints on the latter:

$$\frac{[L]}{[K]} = \left( \frac{\tau_c}{\tau_\varepsilon} \right)^2 > 0; \quad \left( \frac{[K]}{K_\alpha} + \frac{[L]}{L_\alpha} \right) = \frac{4m}{1+3m} > 0 \quad (25)$$

For instance, the first constraint means that the  $\beta$ -phase is simultaneously softer mechanically and chemically or vice versa. If this is not so, then the function  $\tau$  in Eq. (13b), should be changed and terms quadratic in concentration and strain should be included.

### 3.3. Interfacial Adsorption in a Nonlinear System

In order to determine interfacial adsorption at a heterophase interface one has to substitute Eqs. (14a), (14b), (16), and (17) into Eq. (12). Thus for the interfacial adsorption at a heterophase interface one obtains:

$$\begin{aligned} -\frac{1}{\zeta[c]} \Gamma_2^{(1)} = & \left( \frac{2A[\varepsilon][c] + B[\varepsilon]^2}{L_\alpha[c]} \right. \\ & \left. - \frac{A[c]^2 + 2B[\varepsilon][c]}{K_\alpha[\varepsilon]} \right) \frac{\kappa_\eta}{a\eta_\beta^4} \int_{-\infty}^{+\infty} \eta\eta'' dx \\ & + 2 \left( \frac{A[\varepsilon]}{L_\alpha^2[c]^2} - \frac{A[c] + B[\varepsilon]}{L_\alpha K_\alpha[\varepsilon][c]} \right) \kappa_c \\ & \times \int_{-\infty}^{+\infty} cc'' dx + 2 \left( \frac{A[c] + B[\varepsilon]}{L_\alpha K_\alpha[c][\varepsilon]} \right. \\ & \left. - \frac{B[c]}{K_\alpha^2[\varepsilon]^2} \right) \kappa_\varepsilon \int_{-\infty}^{+\infty} \varepsilon\varepsilon'' dx \\ & + O(c''', \varepsilon''', c''\varepsilon'', \eta'''). \end{aligned} \quad (26)$$

Integrals in Eq. (26) can be taken by parts using that all gradients vanish far away from the interface.

To calculate the interfacial segregation we employ the equipartition assumption that

$$\begin{aligned} \kappa_\eta \int_{-\infty}^{+\infty} (\eta')^2 dx &= \kappa_c \int_{-\infty}^{+\infty} (c')^2 dx \\ &= \kappa_\varepsilon \int_{-\infty}^{+\infty} (\varepsilon')^2 dx. \end{aligned} \quad (27)$$

Together with Eq. (9b), the equipartition assumption yields:

$$\begin{aligned} \Gamma_2^{(1)} &= \frac{1}{3} \zeta[c] U \sigma; \\ U &= 2 \left( \frac{A[\varepsilon]}{L_\alpha^2[c]^2} - \frac{B[c]}{K_\alpha^2[\varepsilon]^2} \right) + \frac{1}{a\eta_\beta^4} \\ &\quad \times \left( \frac{2A[\varepsilon][c] + B[\varepsilon]^2}{L_\alpha[c]} - \frac{A[c]^2 + 2B[\varepsilon][c]}{K_\alpha[\varepsilon]} \right). \end{aligned} \quad (28)$$

where  $U$  is a material constant expressed through measurable quantities only. In Ref. [5] we showed that the relative surface entropy, Eq. (10b), vanishes if the gradient energy coefficients  $\kappa$ 's are temperature independent. In this case the Gibbs adsorption theorem takes the form:

$$d\sigma = -\frac{1}{3} \zeta[c] U \sigma d\mu_2; \quad (29)$$

Above we showed that  $[c]$  and  $[\varepsilon]$ , hence  $U$ , are independent of the chemical potentials. Hence, Eq. (29) can be integrated to yield an expression for the surface

energy:

$$\sigma = \sigma_0 e^{-\frac{1}{3} \zeta[c] U \mu_2} \quad (30)$$

The author hopes that Eqs. (28) and (30) may be verified against experimental measurements of the segregation and surface energy at heterophase interfaces in real materials.

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